NAG Toolbox for MATLAB

f11mg

1 Purpose

f11mg computes an estimate of the reciprocal of the condition number of a sparse matrix given an LU factorization of the matrix computed by f11me.

2 Syntax

```
[rcond, ifail] = f11mg(norm_p, n, il, lval, iu, uval, anorm)
```

3 Description

f11mg estimates the condition number of a real sparse matrix A, in either the 1-norm or the ∞-norm:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1 \qquad \text{or} \qquad \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1\left(A^{\mathrm{T}}\right)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function should be preceded by a call to f11ml to compute $\|A\|_1$ or $\|A\|_\infty$, and a call to f11me to compute the LU factorization of A. The function then estimates $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$ and computes the reciprocal of the condition number.

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p – string**

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is to be estimated.

norm_p = '1' or 'O'

$\kappa_1(A)$ is estimated.

norm_p = 'I'

$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **n – int32 scalar**

n, the order of the matrix A.

Constraint: **n** ≥ 0.

3: **il**$(*)$ **– int32 array**

Note: the dimension of the array **il** must be at least as large as the dimension of the array of the same name in f11me.

Records the sparsity pattern of matrix L as computed by f11me.

4: **lval**$(*)$ **– double array**

Note: the dimension of the array **lval** must be at least as large as the dimension of the array of the same name in f11me.

Records the nonzero values of matrix L and some nonzero values of matrix U as computed by f11me.

5: **iu**$(*)$ **– int32 array**

Note: the dimension of the array **iu** must be at least as large as the dimension of the array of the same name in f11me.

Records the sparsity pattern of matrix U as computed by f11me.

6: **uval**$(*)$ **– double array**

Note: the dimension of the array **uval** must be at least as large as the dimension of the array of the same name in f11me.

Records some nonzero values of matrix U as computed by f11me.

7: **anorm – double scalar**

If **norm_p** $=$ '1' or 'O', the 1-norm of the matrix A.

If **norm_p** $=$ 'I', the ∞-norm of the matrix A.

anorm may be computed by calling f11ml with the same value for the parameter **norm_p**.

Constraint: **anorm** ≥ 0.0.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **rcond – double scalar**

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **norm_p** $\neq$ '1', 'O' or 'I',
or **n** < 0,
or **anorm** < 0.0.

ifail $= 301$

Unable to allocate required internal workspace.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f11mg involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{T}}x = b$.

9 Example

```
norm_p = '1';
n = int32(5);
il = [int32(0);
      int32(1);
      int32(2);
      int32(3);
      int32(-1);
      int32(-1);
      int32(1);
      int32(2);
      int32(3);
      int32(5);
      int32(-1);
      int32(0);
      int32(1);
      int32(2);
      int32(3);
      int32(3);
      int32(3);
      int32(0);
      int32(2);
      int32(4);
      int32(6);
      int32(14);
      int32(8);
      int32(2);
      int32(4);
      int32(6);
      int32(8);
      int32(15);
      int32(0);
      int32(2);
      int32(4);
      int32(6);
      int32(8);
      int32(-1);
      int32(2);
      int32(4);
      int32(6);
      int32(8);
      int32(10);
      int32(0);
      int32(4);
      int32(1);
      int32(4);
      int32(2);
      int32(4);
      int32(3);
      int32(4)];
lval = [-2;
      -0.5;
      4;
      0.5;
      2;
```

```
     0.5;
     -1;
     0.5;
     1;
     -1;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0;
     0];
iu = [int32(0);
     int32(0);
     int32(0);
     int32(2);
     int32(3);
     int32(-1);
     int32(0);
     int32(0);
     int32(2);
     int32(3);
     int32(4);
```

```
     int32(1);
     int32(0);
     int32(2);
     int32(1);
     int32(-1);
     int32(-1);
     int32(-1);
     int32(-1)];
uval = [1;
     3;
     1;
     1;
     0;
     0;
     0;
     0];
anorm = 6;
[rcond, ifail] = f11mg(norm_p, n, il, lval, iu, uval, anorm)
```

```
rcond =
    0.0494
ifail =
           0
```